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**3. Goal Programming (Do Not Solve)**

The Tiny Toy Company makes three types of new toys: the tiny tank, the tiny truck, and the tiny turtle. Plastic used in one unit of each is 1.5, 2.0 and 1.0 pounds, respectively. Rubber for one unit of each toy is 0.5, 0.5, and 1.0 pounds, respectively. Also, each tank uses 0.3 pounds of metal and the truck uses 0.6 pounds of metal during production. The average weekly availability for plastic is 16,000 pounds, 9,000 pounds of metal, and 5,000 pounds of rubber. It takes two hours of labor to make one tank, two hours for one truck, and one hour for a turtle. The company allows no more than 40 hours a week for production (priority #1). Finally, the cost of manufacturing one tank is $7, 1 truck is $5 and 1 turtle is $4; a target budget of $164,000 is initially used as a guideline for the company to follow.

* 1. Minimize over-utilization of the weekly available supply of materials used in making the toys and place twice as much emphasis on the plastic (priority #2)
  2. Minimize the under and over-utilization of the budget. Maximize available labor hour usage (priority #3).

Formulate the above decision problem as a single linear goal program. Clearly identify your achievement vector (i.e., hierarchy of priority levels for the goals). Do not solve.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Toy types 🡪 | tiny tank | tiny truck | tiny turtle |  | Total available | p = priority |
| variables | t1 | t2 | t3 |  |  |  |
| Plastic (lb) | 1.5 | 2.0 | 1.0 | <= | 16,000 | p2 |
| Rubber (lb) | 0.5 | 0.5 | 1.0 | <= | 5,000 | p2 |
| Metal (lb) | 0.3 | 0.6 | 0.0 | <= | 9,000 | p2 |
| Labor (hrs) | 2 | 2 | 1 | <= | 40 | p1, p3 |
| Cost & Budget ($) | 7 | 5 | 4 | <= | 164,000 | p3 |

Goal 1: 1.5t1 + 2.0t2 + t3 <= 16,000 (plastic)

Goal 2: 0.5t1 + 0.5t2 + t3 <= 5,000 (rubber)

Goal 3: 0.3t1 + 0.6t2 <= 9,000 (metal)

Goal 4: 2t1 + 2t2 + t3 <= 40 (labor)

Goal 5: 7t1 + 5t2 + 4t3 <= 164,000 (budget)

Priority 1 is saying “don't go over 40”

Priority 2 is saying “2x as much plastic than the other materials”

Priority 3 is saying “go as close as you can to 40”

o = over utilization, u = under utilization

2\*(1.5t1 + 2.0t2 + t3) + u1 - o1 = 16,000

0.5t1 + 0.5t2 + t3 + u2 - o2 = 5,000

0.3t1 + 0.6t2 + u3 - o3 = 9,000

2t1 + 2t2 + t3 + u4 - o4 = 40

7t1 + 5t2 + 4t3 + u5 - o5 = 164,000

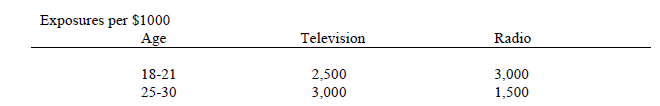
7t1 + 5t2 + 4t3 + u5 - o5 = 0

t1, t2, t3, ui, oi >=0

Lex Min {(o1 + o2 +o3 + o4 + o5), (u6)}

**4. Goal Programming (Do Not Solve)**

XYZ Company is planning an advertising campaign for its new product. The media considered are television and radio. Rated exposures per thousand dollars of advertising expenditure are 10,000 for TV and 7,500 for radio. Management has agreed that the campaign cannot be judged successful if total exposures are under 750,000. The campaign would be viewed as superbly successful if 1 million exposures occurred. In addition, the company has realized that the two most important audiences for its product are persons 18 to 21 years of age and persons 25 to 30 years of age. The following table estimates the number of individuals in the two age groups expected to be exposed to advertisements per $ 1,000 of expenditures:



Management has rank ordered five goals it wishes to achieve, arranged from highest to lowest priorities.

* 1. Achieve total exposures of at least 750,000 persons.
  2. Avoid expenditures of more than $100,000.
  3. Avoid expenditures of more than $70,000 for television advertisements.
  4. Achieve at least 1 million total exposures.
  5. Reach at least 250,000 persons in each of the two age groups, 18-21 and 25-30 years. In addition, management realizes and wishes to account for the fact that the purchasing power of the 25-30 age group is twice that of the 18-21 age group.

Formulate the above decision problem as a single linear goal program. Clearly identify your achievement vector (i.e., hierarchy of priority levels for the goals). Do not solve.

**Formulation:**

Let x be thousands of dollars spent on television and y be thousands of dollars spent on radio

Let n be negative and p be positive deviations

Priority 1: achieve total exposures of at least 750,000 persons

10,000x + 7,500y >= 750,000

10,000x + 7,500y + n1 - p1 = 750,000

Priority 2: avoid expenditures of more than $100,000

x + y <= 100,000

x + y + n2 - p2 = 100,000

Priority 3: avoid expenditure of more than $70,000 on television ads

x <= 70,000

x + n3 - p3 = 70,000

Priority 4: achieve at least 1 million total exposure

10,000x + 7,500y >= 1,000,000

10,000x + 7,500y + n4 - p4 = 1,000,000

Priority 5: reach at least 250,000 people in each of the two age groups

2,500x + 3,000y >= 250,000 (age 18-21)

3,000x + 1,500y >= 250,000 (age 25-30)

2,500x + 3,000y =< 3,000x + 1,500y (both age groups)

3,000x + 1,500y -2,500x - 3,000y >= 0 (re-written the above equation)

2,500x + 3,000y + n5 – p5 = 250,000

3,000x + 1,500y + n6 – p6 = 250,000

3,000x + 1,500y – 2,500x – 3,000y + n7 – p7 = 0

x, y >= 0

Lex Min {(n1 ), (p2 ), (p3 ), (n4 ), (n5 + n6 + n7)}